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NON-STATIONARY TEMPERATURE FIELD IN A TUBE SURROUNDING ENVIRONMENT

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ABSTRACT

In article developed methodology allows to determine the heat flux size from tube surrounding environment to the heat carrier (liquid in tube with a low boiling temperature). Such an operation principle is realized in heat pumps. Heat flow is generally declines over time as the environment temperature falls. In article obtained mathematical expressions allows to determine a reduction of flows, temperature field changes in the environment surrounding the tube and in whole system. Obtained expressions allows to identify and predict the heat pump output changes at the time.

OBJECTIVE OF THE ARTICLE

Heat flux depends on many factors, including the surrounding environment thermophysical characteristics, environmental heat supply possibilities from the farthest surrounding environment regions, as well as temperature changes (environmental long-term cyclic mode change). [1, 2]

The study found non-stationary temperature field calculation expression in an environment that includes cylindrical tube where the coolant is flowing with a constant temperature T_0.

Non-stationary temperature $T_1(r,\tau)$ field in environment setting with a cylindrical tube Fig.1, which filled with a special liquid, at a constant temperature T_0, describes [3] the thermal conductivity equation in polar coordinates:

$$\frac{\partial T_1(r,\tau)}{\partial \tau} = a\nabla^2 T_1(r,\tau), \qquad (1)$$

where a – coefficient of thermal conductivity, $\frac{m^2}{s}$.

On the border $r = r_0$ heat exchange between the liquid and the environment $r \geq r_0$ proceeds in accordance with Newton's Law:

$$\lambda \frac{\partial T_1(r_0;\tau)}{\partial r} = -\alpha[T_0 - T_1(r_0;\tau)], \qquad (2)$$

where α – local heat convection coefficient, $\frac{w}{m^2 K}$;

λ - coefficient of thermal conductivity, $\frac{w}{mK}$;

T_0 - temperature of the liquid inside tube, K.

Temperature at the start moment:

$$T_1(r;0) = T_\infty. \qquad (3)$$

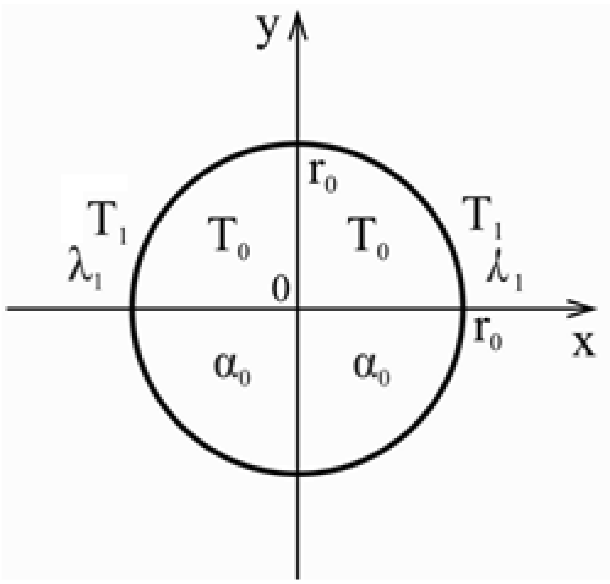

Fig.1 Cylindrical tube cross-section. [4, 5].

At a distance from the axis of the tube temperature is constant:

$$T_1(\infty;\tau) = T_\infty. \quad (4)$$

Dimensionanless variables parameters are entered [3]:

$$\frac{r}{r_0} = \rho; \quad \frac{T_1}{T_0} = \Theta_1; \quad \frac{T_\infty}{T_0} = \Theta_\infty; \quad (5)$$

$$F = \frac{a\tau}{r_0^2}; \quad Bi = \frac{\alpha r_0}{\lambda}.$$

From (1-4) is obtained:

$$\frac{\partial\Theta_1(\rho;F)}{\partial F} = \frac{\partial^2\Theta_1(\rho;F)}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\Theta_1(\rho;F)}{\partial\rho}; \quad (6)$$

$$\frac{\partial\Theta_1(1;F)}{\partial\rho} = Bi[1 - \Theta_1(1;F)]; \quad (7)$$

$$\Theta_1(\rho;0) = \Theta_\infty; \quad (8)$$

$$\Theta_1(\infty;F) = \Theta_\infty. \quad (9)$$

Solved by using the Laplace transformation [3] [6]:

$$\bar{\Theta}_1(\rho,p) = \int_0^{+\infty} e^{pF}\Theta_1(\rho,F)dF. \quad (10)$$

Using the Laplace transformation (10), from (6-9) is obtained:

$$\frac{d^2\bar{\Theta}_1}{d\rho^2} + \frac{1}{\rho}\frac{d\bar{\Theta}_1}{d\rho} - p\bar{\Theta}_1 = -\Theta_\infty; \quad (11)$$

$$\frac{d\bar{\Theta}_1(1;p)}{d\rho} = -Bi\left[\frac{1}{p} - \bar{\Theta}_1(1;p)\right]. \quad (12)$$

Equation (11) is the inhomogeneous Bessel differential equation [7]. The general solution expressed in [8] with the first and second type Mcdonald (Bessel) functions $I_0(z)$ and $K_0(z)$:

$$\bar{\Theta}_1(\rho;p) = C_1 I_0(\rho\sqrt{p}) + C_2 K_0(\rho\sqrt{p}) + \frac{\Theta_\infty}{p}. \quad (13)$$

Solution must be final, where $\rho \to \infty$. So that $\lim_{z\to\infty} I_0(z) = \infty$, it must be $C_1 = 0$.

Therefore, from (13) obtained:

$$\bar{\Theta}_1(\rho;p) = C_2 K_0(\rho\sqrt{p}) + \frac{\Theta_\infty}{p}. \quad (14)$$

Using the boundary conditions (12) and equation [8], if $K_0'(z) = -K_1(z)$, we obtain:

$$C_2 K_1(\sqrt{p})\sqrt{p} = -Bi\left[\frac{1}{p} - C_2 K_0(\sqrt{p}) - \frac{\Theta_\infty}{p}\right], \quad (15)$$

from witch is expressed constant:

$$C_2 = \frac{Bi(1-\Theta_\infty)}{p[BiK_0(\sqrt{p}) - \sqrt{p}K_1(\sqrt{p})]}. \quad (16)$$

Inserting the calculated constant into the expression (14) we can obtain the searched temperature picture:

$$\bar{\Theta}_1(\rho;p) = \frac{1}{p}\left[\Theta_\infty + \frac{Bi(1-\Theta_\infty)K_0(\rho\sqrt{p})}{BiK_0(\sqrt{p}) - \sqrt{p}K_1(\sqrt{p})}\right]. \quad (17)$$

Original temperature expressed in [3], [6] as follows:

$$\int_{\gamma-j\omega}^{\gamma+j\omega} \frac{K_0(\rho\sqrt{p})e^{pF}}{p[BiK_0(\sqrt{p}) - \sqrt{p}K_1(\sqrt{p})]}dp. \quad (18)$$

Integral along a straight line $Re\,p = \gamma$, where p is a complex number $p = \gamma + j\omega$ can be expressed as [3], [6] the integrals along the straight line AB and the circle $p = Re^{j\varphi}$ bows BB'F and CA'A, along the cut line Fig.2 EF upper edge and lower edge of the CD, and in small circle line $p = re^{j\varphi}$, which together form the closed contour.

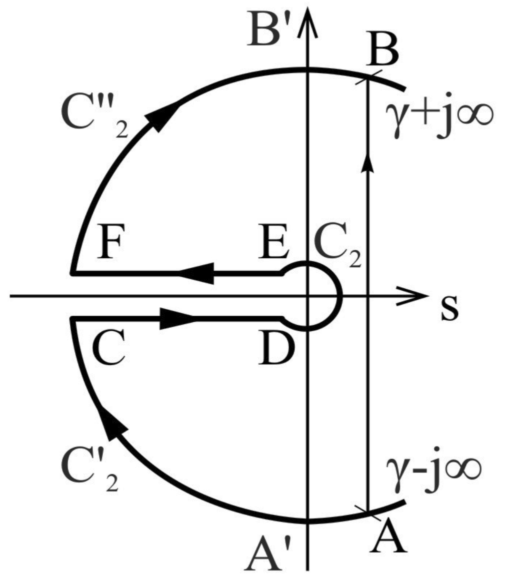

Fig.2 Integration circuit. [4, 5].

The integral over the straight edge AB is equal to sum of integrals over all other parts of the circuit, as underintegral function is inside of analytical expression. Calculating the integrals, where $R \to \infty$ and $r \to 0$ are obtained integral required for formula (18).

$$\int_{\gamma+j\omega}^{\gamma+j\omega} \frac{K_0(\rho\sqrt{p})e^{pt}}{p[BiK_0(\sqrt{p})-\sqrt{p}K_1(\sqrt{p})]}dp =$$

$$= \lim_{R\to\infty}\left(\int_{BB'F} + \int_{FE} + \int_{DC} + \int_{CA'A}\right) = \lim_{\varepsilon\to 0}\oint_{C_r} \quad (19)$$

Integrals along the major arc of a circle BB'F and CA'A if $R \to \infty$, equal to zero.

On the small circle $p = re^{j\varphi}$; $dp = re^{j\varphi}jd\varphi$. We can calculate:

$$\lim_{r\to 0}\int_{C_r} \frac{K_0(\rho\sqrt{p})e^{pt}}{p[BiK_0(\sqrt{p})-\sqrt{p}K_1(\sqrt{p})]}dp =$$

$$\lim_{r\to 0}\int_0^{2\pi} \frac{K_0(\rho\sqrt{re^{j\varphi}})e^{re^{j\varphi}F}}{re^{j\varphi}[BiK_0(\sqrt{re^{j\varphi}})-\sqrt{re^{j\varphi}}K_1(\sqrt{re^{j\varphi}})]}re^{j\varphi}jd\varphi \sim$$

$$\lim_{p\to 0}\int_0^{2\pi} \frac{ln\frac{2}{\gamma r\sqrt{p}}}{Bi\,ln\frac{2}{\gamma\sqrt{p}}-\sqrt{p}\frac{1}{2}\frac{2}{\sqrt{p}}}jd\varphi \sim$$

$$\lim_{p\to 0}\int_0^{2\pi} \frac{1}{Bi}jd\varphi \sim \frac{2\pi j}{Bi}. \quad (20)$$

Let us donate the line EF [3]:

$p = u^2e^{j\pi} = -u^2; \Rightarrow dp = ue^{j\pi}du; \sqrt{p} = ue^{j\frac{\pi}{2}} = ju$.

From (19) obtained:

$$\lim_{R\to 0}\int_{EF} = \int_0^{+\infty} \frac{K_0(\rho\sqrt{p})e^{pF}}{p[BiK_0(\sqrt{p})-\sqrt{p}K_1(\sqrt{p})]}dp =$$

$$\int_0^{+\infty} \frac{K_0(ju\rho)e^{-u^2F}2ue^{j\pi}du}{u^2e^{j\pi}[BiK_0(ju)-juK_1(ju)]} =$$

$$2\int_0^{+\infty} e^{-u^2F}\frac{K_0(ju\rho)}{BiK_0(ju)-juK_1(ju)}\frac{du}{u} =$$

$$2\int_0^{+\infty} e^{-u^2F}\frac{-J_0(\rho u)+j-Y_0(\rho u)}{-BiJ_0(u)-uJ_1(u)+j[BiY_0(u)+uY_1(u)]}\frac{du}{u}. \quad (21)$$

Here, using the relationship:

$$K_v\left(ze^{\pm j\frac{\pi}{2}}\right) = K_v(\pm jz) =$$

$$\pm\frac{\pi}{2}je^{\pm jv\frac{\pi}{2}}[-J_v(z)\pm Y_v(z)], \quad (22)$$

where $J_v(z)$ and $Y_v(z)$ is the first and second kind Bessel's functions.

In underintegral expression will find real and imaginary parts. Therefore, get rid of imaginarily part of the denominator by multiplying the numerator and denominator by the denominator complex related expression. We can introduce terms:

$$\varphi(u) = BiJ_0(u) + uJ_1(u); \quad (23)$$

$$\Psi(u) = BiY_0(u) + uY_1(u); \quad (24)$$

$$\varphi_1(u) = J_0(\rho u)[BiJ_0(u)+uJ_1(u)] +$$

$$+Y_0(\rho u)[BiY_0(u)+uY_1(u)]; \quad (25)$$

$$\Psi_1(u) = -Y_0(\rho u)[BiJ_0(u)+uJ_1(u)] +$$

$$+J_0(\rho u)[BiY_0(u)+uY_1(u)]. \quad (26)$$

Using expressions (23-26), the integral 21, can be written as follows:

$$\int_{EF} = 2\int_0^{+\infty} e^{-u^2F}\frac{\varphi_1(u)+j\Psi_1(u)}{\varphi^2(u)+\Psi^2(u)}\frac{du}{u}. \quad (27)$$

Let us denote on the line CD [3]:

$p = u^2e^{j\pi} = -u^2; \Rightarrow dp = 2ue^{-j\pi}du; \sqrt{p} = ue^{-j\frac{\pi}{2}} = -ju$

Analogically, as (21-27) we can get:

$$\int_{CD} = 2\int_0^{+\infty} e^{-u^2F}\frac{\varphi_1(u)+j\Psi_1(u)}{\varphi^2(u)+\Psi^2(u)}\frac{du}{u}. \quad (28)$$

Inserting the calculated integrals formulas 18 and 19, acquires temperatures expression in environment surrounding tube:

$$+\frac{4j}{2\pi j}\int_0^{+\infty} e^{-u^2F}\frac{\Psi_1(u)}{\varphi^2(u)+\Psi^2(u)}\frac{du}{u}. \quad (29)$$

or

$$\Theta_1(\rho, F) == 1 + \frac{2}{\pi}\int_0^{+\infty} e^{-u^2 F}\frac{\Psi_1(u)}{\varphi^2(u)+\Psi^2(u)}\frac{du}{u}. \quad (29)$$

Using the Laplace transform expression was derived temperature field calculation expressions in tube surrounding environment.

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