Non-isothermal flow of a non-Newtonian two-phase media in the area with a permeable surface

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The study was supported by The Ministry Of education and science of Russia, project 14.B37.21.0644

ABSTRACT

Non-isothermal flow of two-phase media with complex rheological equation of state in areas with permeable surfaces are found in many industrial processes. In connection with this work is to construct a mathematical model non-isothermal laminar flow of non-Newtonian two-phase media with their filtering through the permeable surface of the workers and the numerical scheme for calculating them. Equations of mechanics of multiphase media stored in an arbitrary orthogonal coordinate system associated with the flow region. To solve this problem a method adapted surfaces of equal costs. We consider two-dimensional and axisymmetric flow and made specific numerical calculations of thin-layer flow on flat surfaces and rotating and axially symmetric flat channels and pipes for media with rheological equation of state Ostwald de Vilya. The effect of the filtering process, and non-isomorphic thermals on hydrodynamic conditions in the flow domain. The results represent a great practical interest for various technological applications: Process mixture solution u dispersion, separation and condensation of two-phase media in the filter apparatus, realization of the intense heat, boundary layer control, etc.

INTRODUCTION

In many processes are widely used devices with permeable work surfaces with multi-phase working media. Design and calculation of the efficiency of such devices are associated with the modeling and solving internal or external, and mixed problems termogidrodynamics of multiphase flow in their work sites that require knowledge of the thermohydrodynamic characteristics of streams, given the dependence of the rheological and physicalmechanical properties of the fluid temperature. Isothermal flow of two-phase media in areas with permeable surfaces of the investigated in several studies, such as in [1-3]. There are a number of studies on the thermohydrodynamic situation in areas with porous walls for Newtonian liquids and gases, for example [4,5]. Thermohydrodynamics rheology of complex multiphase flows in areas with permeable surfaces, taking into account the temperature dependence of the properties of the environment is not well understood.

THE THEORETICAL PAR

Laminar thin-considered flat and axisymmetric flow to two-phase non-Newtonian media or permeable surfaces in the channels and pipes with the complex shape of the liquid phase through the filtering surface of the flow in the orthogonal coordinate system whose coordinate \( x_2 = \text{const} \) surface coincides with the surface area, and the coordinate lines (surfaces) \( x_1 = \text{const} \) are normal family to her. Then, taking into account the importance of the analysis of members simplified dimensional equations of conservation of mass, momentum and energy for the two-phase media can be written as [6]
\[ \frac{\partial (H_2H_3 \rho U_1)}{\partial x_1} + \frac{\partial (H_2 H_3 \rho V_1)}{\partial x_2} = 0, \quad (2) \]

\[ \rho_1 \left( \frac{U_1}{H_1} \frac{\partial U_1}{\partial x_1} + \frac{V_1}{H_2} \frac{\partial U_1}{\partial x_2} + \frac{U_1V_1}{H_1 H_2} \frac{\partial H_1}{\partial x_2} \right) = - \frac{\alpha_1 \partial p}{H_1} + \tau_{12} - F_{12}, \quad (3) \]

where \( p \) - pressure, \( \tau_{12} \) - tangential force at the interface, \( F_{12} \) - force vector of interfacial interaction; \( \alpha_1 \) - specific heat, \( \beta \) - heat transfer coefficient, \( d \) - wall thickness, \( h(x_1) \) - film thickness.

The equations of motion (2) - (9) form a system of nonlinear differential equations, which are not integrated in quadratures. In this paper, to solve this system, we adapted our method of surfaces of equal cost [7]. In accordance with this method we introduce in the field of suspension flow streamlines 

\[ \psi_k = \psi_{k_1}(x_1) = x_{k,1} \quad \text{and present the components of the velocity of i-th phase for the k-th layer in the form:} \]

\[ U_i^k = U_i(x_1, y_k^i(x_1)), \quad V_i^k = V_i(x_1, y_k^i(x_1)), \quad T_i^k = T_i(x_1, y_k^i(x_1)) \]

where \( k = 1, N \), \( N \) - number of current lines. And the line \( y_1 \) coincides with the surface flow, and the line of \( y_N \) with free surface. The authors reduce the problem of the development of the flow layer of the suspension to the numerical determination of the velocity field, temperature and streamlines.

Denote the value of changes in flow lines of the first phase between \( y_k \) and \( y_{k+1} \) through \( \Phi^k_1(x_1) = 0 \). By definition

\[ \frac{d}{H_1 dx_1} \int_{y_k}^{y_{k+1}} \alpha_i Z U_i H_2 dx_2 = \Phi^k_1(x_1) \quad (11) \]

In the absence of mass transfer variation of this flow may occur due to filtration of fluid through the permeable surface. Assume that the coordinate system is chosen so that the values of \( \alpha_i, h, U_i, V_i \) Assume that the coordinate system is chosen so that the values of \( x_3 \) and denote \( Z = H_2(x_3 = x_{3,0}) \). Then the integral condition of conservation of the solid phase for arbitrary section can be written as:

\[ \int_{x_{3,0}}^{x_{3,1}} \alpha_i Z U_i H_2 dx_2 + \int_{x_{3,1}}^{x_{3,2}} \alpha_i Z V_0 H_i dx_1 = Q_{iw} \]

Differentiating this relation with respect \( x_1 \) and after comparing the results with the differentiation (11) authors obtain:

\[ \Phi^k_1(x_1) = \alpha_i Z V_0 \delta^k, \quad k = 1, N \] \quad (12)

Applying the Leibniz rule, taking into account Eq. (2) calculate the integral Eq. (11)

\[ \Phi^k_1 = \alpha_i Z (V_i^k U_i^k - U_i^k V_i^k) - \alpha_i Z (V_i^k U_i^k H_2 dy_k^i - U_i^k V_i^k H_i dx) \]

Hence, taking into account the kinematic condition on the free surface and the relations Eq. (12), authors obtain

1. Thin-layer flow on permeable surface

The system of equations (2) - (9) and the filtering equation must be solved with the following boundary and initial conditions:

at \( x_2 = 0, P_0 = P_a \); \quad (10.1)

at \( x_3 = d: P = P_a, U_1 = 0, V_0 = \alpha_i V_i, T_1 = T_{CT} \); \quad (10.2)

at \( x_2 = h_i = P = P_a, \tau_{12} = 0; \quad (10.3) \)
\[ \alpha_i Z (v_i^1 - u_i^1 d_y^k / h_i dx_1) = \Phi^k \delta^k, \quad k = 1, N. \] (13)

Derivatives of the independent variable, \( x_i \), have the form:
\[ \frac{d \Theta}{h_i dx_1} = \frac{\partial \Theta}{h_1 \partial x_1} + \frac{\partial \Theta}{h_2 \partial y_1} / h_1 dx_1. \] (14)

Replacing partial derivative \( \partial U_i / \partial x_i \) according to Eq. (14) and taking into account relations Eq. (13) authors write the Eq. (3) for \( k \) layer
\[ \rho_i U_i^k d U_i^k / h_i dx_1 = -\alpha_i \partial \rho_i + \rho_i U_i^k V_i^k / h_1 \partial x_1 + \rho_i F_i^k. \] (15)

After Integration over the interval \([y_k, y_{k+1}]\) sum of Eqs. (4) and (8) we obtain
\[ P_{k+1} - P_k = M_k, \quad k = 1, N - 1, \] where
\[ M_k(x_i) = \int (x_1, x_2) dx_2, J(x_1, x_2) = H_2 F_2 \]
\[ + \rho_i U_i^k j + \rho_i U_i^k H_i / h_1 dx_1. \]

Transforming the result to the convenient form and differentiating along the longitudinal coordinate, authors obtain
\[ \frac{d P_k}{dx_1} = \sum_{j=1}^{N-1} d M_k / dx_1, \quad k = 1, N - 1. \] (16)

Replacing in Eq. (15) \( \partial P_k / \partial x_1 \) and Eq. (16), and using the sum of Eqs. (4) and (8), to determine the rate of continuous phase, authors obtain
\[ \rho_i U_i^k j + \rho_i U_i^k H_i / h_1 dx_1 = \alpha_i \sum_{j=1}^{N-1} d M_k / dx_1 + \alpha_j J(x_1, x_2) d y_k / h_i dx_1 \]
\[ + \rho_i U_i^k j + \rho_i U_i^k H_i / h_1 dx_1 / h_1 dx_1. \] (17)

Similarly, authors can obtain equations for the dispersed phase. Omitting intermediate findings, we present the final expression for the velocity distribution
\[ U_i^k = \sum_{j=1}^{N-1} d M_k / dx_1, \quad k = 1, N - 1. \] (18)

If the values of inclusion size and density difference of phases are small, the relative motion of phases may be negligible. Then you can use the quasi-homogeneous flow model. In the quasi-homogeneous approximation of the current lines are introduced quite clear for some effective medium with variable characteristics in the longitudinal coordinate \( \rho(x_i), \mu(x_i) \). Transformed equation of motion of the effective medium can be obtained by adding the Eqs. (17) and (28)
\[ \rho U_i^k d U_i^k / h_i H_i dx_1 = \sum_{j=1}^{N-1} d M_k / dx_1 + J(x_1, x_2) d y_k / h_i dx_1 \]
\[ \rho UV_i^k / h_i H_i \partial x_1 + \tau_{12}^k + \rho_i F_i. \] (19)

The equation for the surfaces of equal flow rate is determined from the Eq. (11). Why do authors represent the integral over one of the formulas of numerical integration and then differentiate obtain difference equation in \( x_i \). If authors integrate Eq. (11) by the trapezoidal formula, the equation for determining the current lines has the form.
\[ \frac{d y_{k+1}}{dx_1} = \frac{d y_k}{dx_1} + 2 H_2 \delta k / \Delta_k, \quad y_{k+1}, y_k / \Delta_k / dx_1, k = 2, N. \] (20)

where \( \Delta_k = (\alpha_i H_2 Z U_i) + (\alpha_i H_2 Z U_i)_{k+1} \).

To calculate the viscous term grid solutions can be represented as a series expansion in a complete system of basis functions satisfying the boundary conditions Eq. (10)
\[ \sum_{j=1}^{N} A_j(x_1) U_j(x_1), \quad T^k(x_1) = \sum_{j=1}^{N} A_j^T T^k(x_1). \] (21)

The system of basic functions can be chosen as [7]
\[ U_{j+1}(x_i) = \left( j + \frac{1}{j} - \frac{1}{j} \right) \eta_j(x_i), \]
\[ \text{where} \quad \eta_j(x_i) = \frac{y_j(x_i)}{y_k(x_i)}; \quad j = 1, N; \quad k = 1, N. \]

The system of basic functions for temperature can be chosen as:
\[ T^k(x_1) = \left( y^k(x_1) \right)^J \]

The authors require that the velocity and temperature determined from Eq. (21), coincided with the \( U_i^k(x_1) \) and \( T^k(x_1) \) on lines \( y^i(x_1) \). Then for determination the coefficients \( A_j(x_1) \) and \( A_j^T(x_1) \) obtain a system of algebraic equations:
\[ \sum_{j=1}^{N} A_j(x_1) U_j[y_j(x_1)] = U_i^k(x_1), \quad T^k(x_1) = \sum_{j=1}^{N} A_j^T(x_1) T^k(x_1), \quad k = 1, N. \] (21.1)

Having determined the value \( A_j(x_1) \) of the system of equations, we can calculate the “viscous” term \( \tau_{12} \) on the respective flow lines. Having determined the value \( A_j^T(x_1) \) using the expansion (21.1) can be evaluated \( T^k \) at the appropriate temperature streamlines. The system (2) - (9), (19) - (20) and boundary conditions (10) constitute a closed
system of equations whose solution with known right-hand side can be obtained in one of numerical methods for solving ordinary differential equations.

In the equations of motion (17) - (19) recorded for any surface equal spending $y_k$ is present term $\alpha_i \sum_{l=2}^{N} dM_{l,1} \text{d}x_l$, which has an undefined value $\text{d}y_k / \text{d}x_l$. At the same time, the derivatives of $\text{d}y^\prime_l / \text{d}x_l$ are defined by the recurrence relation (20) upwards from $\text{d}y^\prime_l / \text{d}x_l$. Therefore, to calculate the right-hand side of (17) - (19) it is necessary to use the sweep. Find explicit expressions for the coefficients progonochny advisable to perform after specifying the flow domain and the definition of the Lyame coefficients $H_1, H_2, H_3$.

Example. The flow over a flat surface.

Let layer heterogeneous medium flows along an inclined plane. We choose a Cartesian coordinate system with $x_1 = x, x_2 = y, x_3 = z$ coefficients Lyame $H_1=1, H_2=1, H_3=1$. Then $F_i = g \sin \varphi, F_2 = g \cos \varphi, J(x,y) = \rho F_2$, $M_k(x) = \rho F_2 (y_k - y_{k-1}), Z = 1$.

Here $g$ - acceleration of gravity, m/s², $\varphi$ - angle plane.

Differential Eqs. (5),(9), (19) - (20) for a plane of unit width $(Z = 1)$ take the form

$$\rho U_{i,k} \frac{dU_{i,k}}{dx} = g \cos \varphi \left( \frac{y_N - y_k}{\rho} \frac{d\rho}{dx} + \rho \frac{dy_N}{dx} \right) +$$

$$+ m \frac{\partial}{\partial y} \left( \frac{U_{i,k}}{\rho} \right)^n + \rho g \sin \varphi, \quad \frac{dy_{k+1}}{dx} = \frac{dy_k}{dx} + \frac{2Y_0 \delta^k}{\alpha_i (U_{i,k} + U_{i,k+1})}$$

$$- \frac{y_{k+1} - y_k}{\alpha_i (U_{i,k} + U_{i,k+1})} \frac{d}{dx} \left[ \alpha_i (U_{i,k} + U_{i,k+1}) \right].$$

$$U_{i,k} \frac{\partial U_{i,k}}{\partial x} + V_{i,k} \frac{\partial U_{i,k}}{\partial y} = \alpha_i \frac{\partial T_i}{\partial y^2},$$

Here are $\alpha_i$ - thermal diffusivity.

Introduce dimensionless variables of the system by substituting

$$x = H \text{Re} x_1, y = H H_2 U_{y1}, V_y = U_{y1}, V_0 = \text{Re} U_{y1}, P = \rho U_{y1}^2 \text{Re}, \text{Re} = H^2 \frac{U_{y1}^2 \rho_H}{m_H},$$

This system in dimensionless variables takes the form (dash omitted):

$$U_{i,k} \frac{dU_{i,k}}{dx} = \frac{(y_N - y_k)}{\rho} \frac{d\rho}{dx} + \frac{dy_N}{dx} \frac{\cos \varphi}{\rho},$$

$$+ \frac{\rho}{Y_0 \alpha_i} \frac{\partial U_{i,k}}{\partial \xi} \frac{\partial U_{i,k}}{\partial \xi} = \frac{\alpha_i \delta T_i}{\partial \xi^2},$$

$$\frac{dy_{k+1} - y_k}{\alpha_i (U_{i,k} + U_{i,k+1})} \frac{d}{dx} \left[ \alpha_i (U_{i,k} + U_{i,k+1}) \right].$$

$$U_{i,k} \frac{\partial U_{i,k}}{\partial x} + V_{i,k} \frac{\partial U_{i,k}}{\partial y} = \frac{\alpha_i \delta T_i}{\partial y^2},$$

Here $F_i = V_{i,2}^2 / F_i - F_i - F_i / a$ - Pelet number; $Y_0 = \rho U_{y1}^2 / m$ - Reynolds number; $j_{i,k}$ - the size of the flow in the direction of $\zeta^i, \delta_{i,k}^k$ - Kronecker delta.

To calculate the right-hand side sweep method the first two equations of the resulting system to the form

$$y_{k+1} - y_k + S_k U_{i,k} + S_k U_{i,k}^2 = E_k, U_{i,k} = D_k + C_k y_{N,k},$$

$$S_k = \frac{y_{k+1} - y_k}{\alpha_i (U_{i,k} + U_{i,k+1})}.$$

Here and below, for the convenience of room surfaces of equal cost and speeds, written as a subscript, and the primes denote derivatives with respect to the dimensionless longitudinal coordinate.

The authors represent the unknown function $y_{N,k}$ as a sweep method ratio:

$$y_{N,k} = A_k y_{N,k} + B_k, \quad (24)$$

and substitute into Eq. (23). After simple transformations authors obtain explicit expressions a sweep method coefficient in the form of the recurrence

$$A_k = A_{k-1} - S_k C_{k-1} - S_k C_k,$$

$$B_k = B_{k-1} - S_k D_{k-1} - S_k D_k + E_k, k = 2, N; \quad (25)$$

$$A_2 = -S_2 C_2, \quad B_2 = E_2 - S_2 D_2.$$

At $k = N$ from Eq. (24) authors obtain $y_{N,k} = B_k / (1 - A_N)$.

In the quasi-homogeneous approximation, when

$$U_{i,k} = U_i, V_i = V_i = V, T_i \approx T_i = T$$

system (2)-(9) in the case of flow down an inclined plane is written as:
\[ \frac{\partial (\rho U)}{\partial x_1} + \frac{\partial (\rho V)}{\partial x_2} = 0, \]  
(26)

\[ \rho \left( U \frac{\partial U}{\partial x_1} + V \frac{\partial U}{\partial x_2} \right) = \alpha \frac{\partial \rho U}{\partial x_1} + \tau_{12} + \rho F, \]  
(27)

\[ \alpha \frac{\partial \rho U}{\partial x_2} + \rho F = 0, \]  
(28)

\[ \rho e_p \left( U \frac{\partial \rho}{\partial x_1} + V \frac{\partial \rho}{\partial x_2} \right) = \frac{\partial}{\partial x_2} \left( \lambda \frac{\partial \rho}{\partial x_2} \right). \]  
(29)

The system (26)-(29) is solved with the following boundary and initial conditions:

at \( x_2 = 0 \) : \( P_0 = P_a \); \( U = 0 \); \( T = T_{cv} \); \( V_0 = \alpha_1 V \); \( \tau_{12} = 0 \),

\[ \frac{\lambda}{\partial x_2} \rho = \beta [T(h) - T_{air}], \]  
(30.2)

at \( x_1 = x_{iin} \) : \( \alpha = \alpha_{in} \); \( U = U_{in}(x_2) \); \( V = V_{in}(x_2) \); \( T = T_{in}(x_2) \); \( h = H \).  
(30.3)

For this example, a program was developed and numerical calculations, the results of which are shown below.

### The results of computer simulation

Processes were calculated heating and cooling medium, both from the wall and from the free surface of the film with the filtration of the liquid phase through the permeable surface.

Fig. 1 shows the temperature change on the flow lines for movement of the medium on the "hot" wall. Changing the temperature begins from the lower layer and transmitted upward. Upon reaching the thermal boundary layer of the free surface of the temperature change occurs at the free surface film. Filtration of the solid phase through permeable surface reduces the thickness of the heterogeneous medium. This changes the position of the lines of equal surface charges. Fig. 2 shows the temperature change of the flow lines in the flow of a heterogeneous environment on an impermeable surface.

![Fig. 1](image1.png)

**Fig. 1.** The temperature profiles on the lines of equal surface flow at \( h_{in} = 10^{-2} m \).

2. **Pressure flow in channels and pipes with permeable walls**

In this case, the system of equations (2) - (9) and the equation of filtration instead of the boundary conditions (10) are solved with the following boundary conditions:

at \( x_2 = 0 \) : \( P_0 = P_a \); \( U = 0 \); \( T = T_{cv} \); \( V_0 = \alpha_1 V \); \( \tau_{12} = 0 \),

\[ \frac{\lambda}{\partial x_2} \rho = \beta [T(h) - T_{air}], \]  
(30.2)

at \( x_1 = x_{iin} \) : \( \alpha = \alpha_{in} \); \( U = U_{in}(x_2) \); \( V = V_{in}(x_2) \); \( T = T_{in}(x_2) \); \( h = H \).  
(30.3)

The problem (2) - (9) under the boundary conditions (31) is solved similarly by using the scheme outlined above problem (2) - (10).

In solving the problems (2) - (9), (10) and (2) - (9), (31) in the first approximation assumed that thermal phase parameters than the effective viscosity of the carrier phase change slightly depending on temperature, and to account for the temperature dependence of the effective viscosity was used polynomial approximation.

\[ u_{in} = 10^{-2} m/sec, \quad Re_u = 0.16, \quad T_{fluid} = 30^\circ C, \]

\[ T_w = 40^\circ C, \quad T_{air} = 30^\circ C, \quad k = 1.27 \cdot 10^{-12} m. \]

![Fig. 2](image2.png)

**Fig. 2.** The temperature profiles on the lines of equal surface flow at \( h_{in} = 10^{-2} m \),

\[ u_{in} = 10^{-2} m/sec, \quad Re_u = 0.16, \quad T_{fluid} = 30^\circ C, \]

\[ T_w = 40^\circ C, \quad T_{air} = 30^\circ C, \quad k = 0 m. \]
CONCLUDE

Been adapted method of surfaces of equal cost to the process flow and filtering of a two-phase fluid through a permeable surface in the non-isothermal conditions, the numerical calculations are performed in the quasi-homogeneous approximation. Constructed a mathematical model to calculate the processes of non-isothermal flow of a two-phase medium in areas with permeable surfaces at different thermal regimes and forms the basis for the formulation of optimal design of process equipment units.

References


